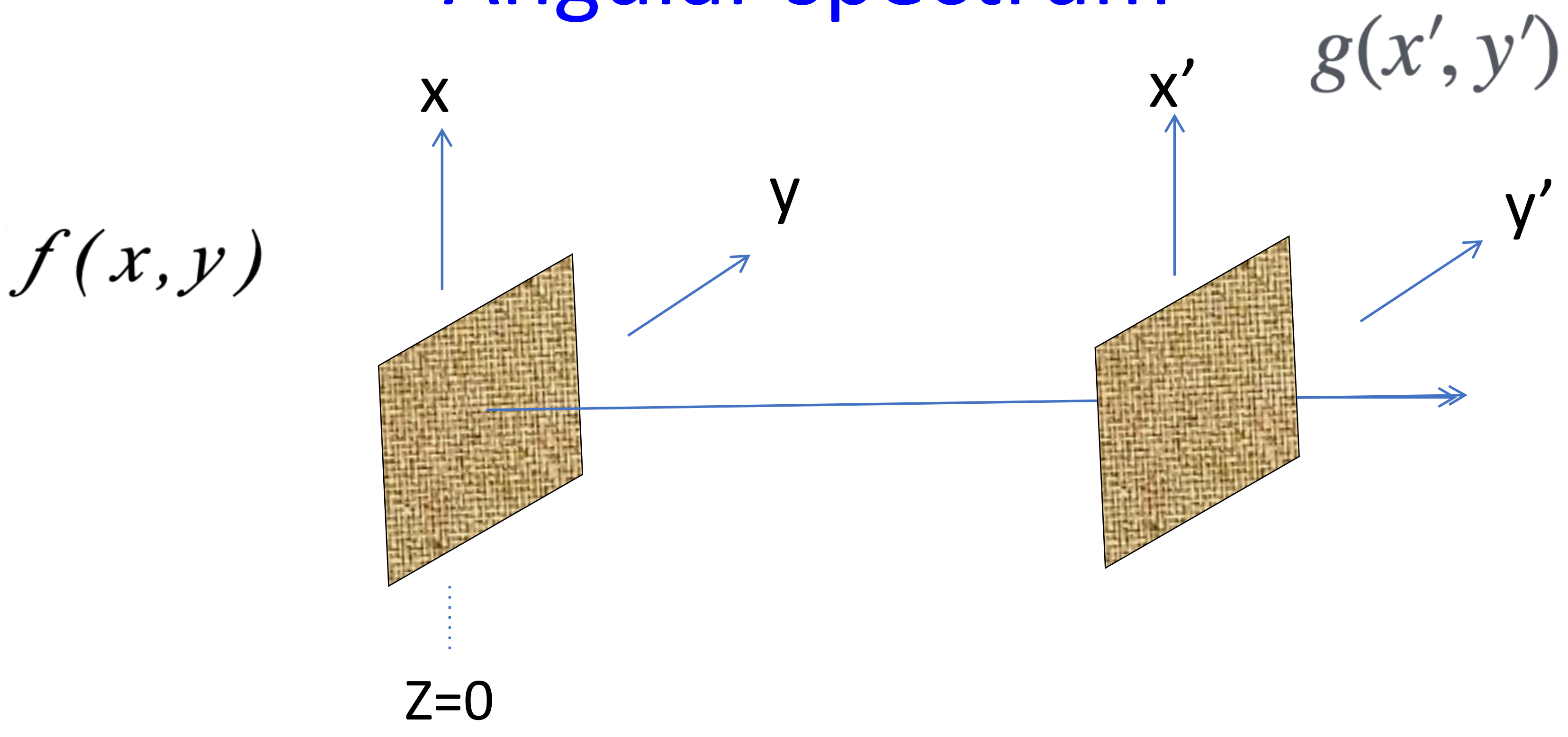
Computational Optical Imaging

Lecture 2

Outline

- Last time: Angular Spectrum Method (ASM) and 2D FTs
- Fresnel diffraction
- Near field and NSOM (near field optical microscopy)
- Far field
- Exercise #1
- Gratings
- Talbot imaging
- Bessel beam and Light sheet imaging
- Exercise #2

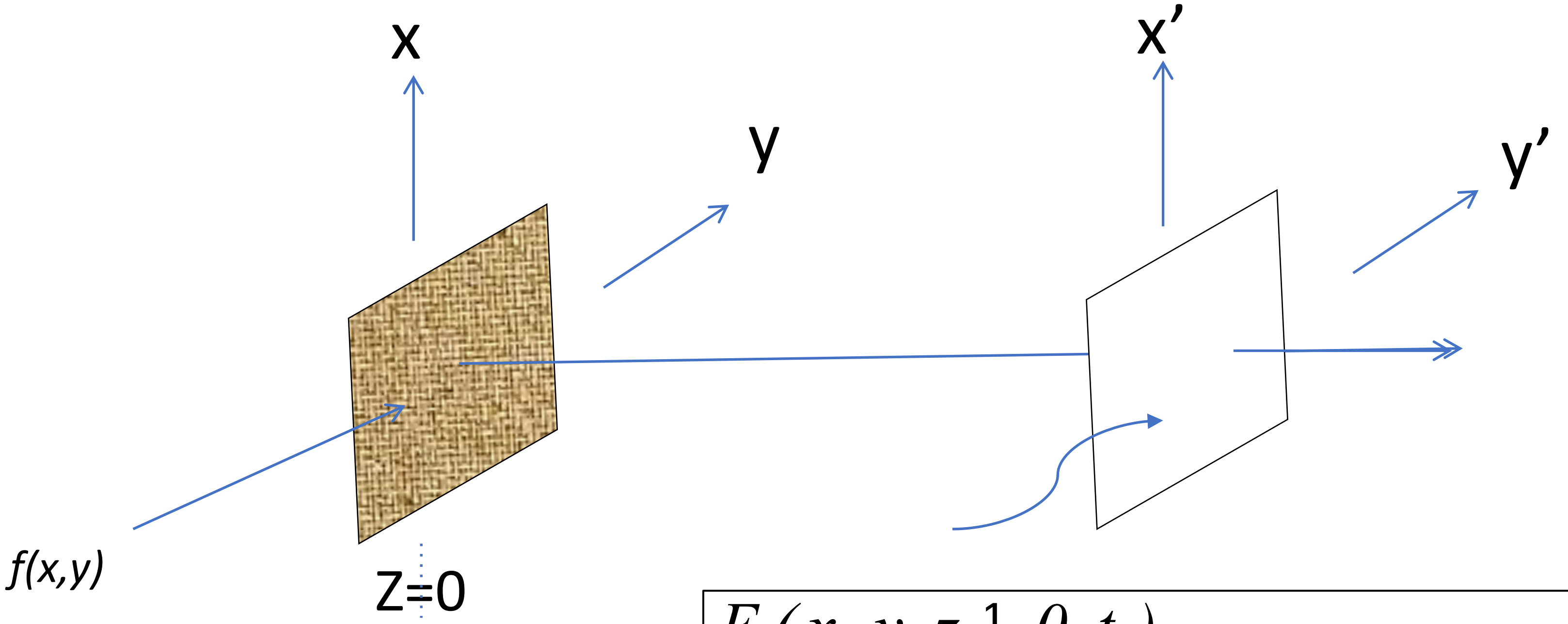
Angular spectrum



3 steps

1. Take the 2D Fourier transform of f(x,y)
2. Multiply each spatial frequency component by its eigenvalue
3. Take the inverse 2D Fourier transform to get g(x',y')

Angular spectrum



$E(x,y,z=0,t)=Af(x,y)e^{j\omega t}$

$f(x,y) \xleftrightarrow{\text{2D FT}} F(u,v)$

$$E(x,y,z \neq 0,t)$$
$$= e^{j\omega t}\iint F(u,v)e^{-j\left(2\pi ux+2\pi vy+\sqrt{k^2-(2\pi u)^2-(2\pi u)^2}\,z\right)}\,du\,dv$$

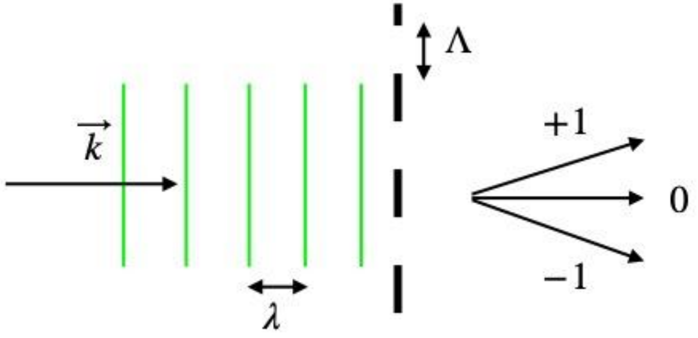
$$E(x,y,z=0,t)=Ae^{j\omega t}f(x,y)=Ae^{j\omega t}(2\pi)^2\iint F(u,v)e^{-j2\pi(ux+vy)}\,du\,dv$$

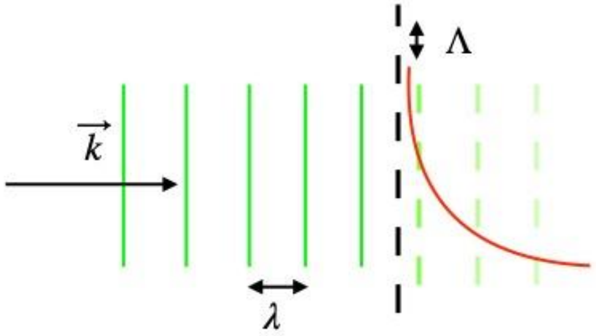
$$k_x=2\pi u \quad k_y=2\pi v \quad k_z=\sqrt{k^2-k_x^2-k_y^2}=\sqrt{k^2-(2\pi u)^2-(2\pi v)^2}$$

Evanescent Waves

$$k_x^2 + k_y^2 + k_z^2 = \omega^2\mu\epsilon = (\frac{2\pi}{\lambda})^2 = k^2$$

$$k_z = \sqrt{k^2 - (k_x^2 + k_y^2)}$$





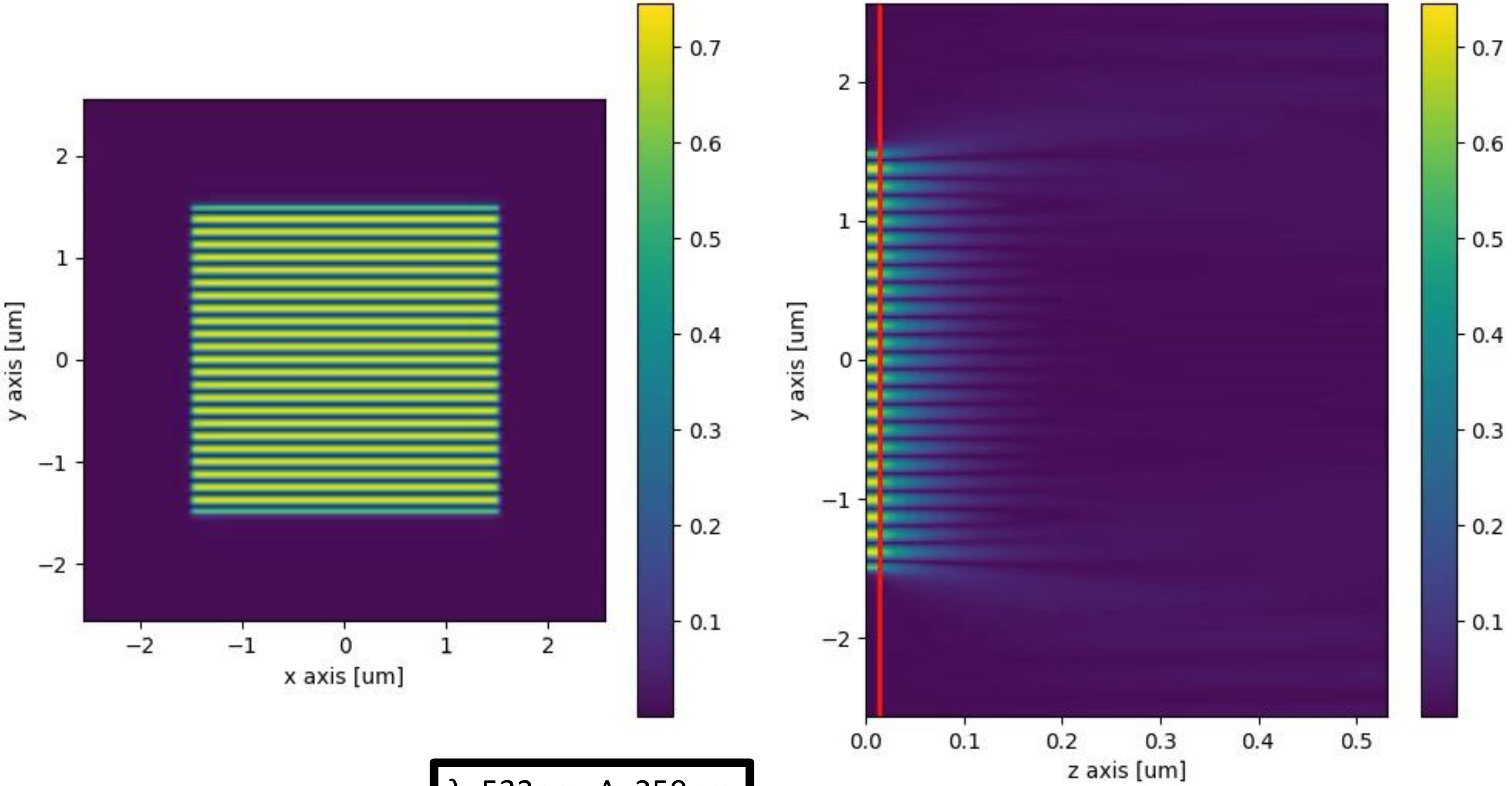
$$k_x = \frac{2\pi cos\theta_x}{\lambda} = \frac{2\pi}{\Lambda}$$

$$u = \frac{1}{\Lambda}$$

Exponential Decay

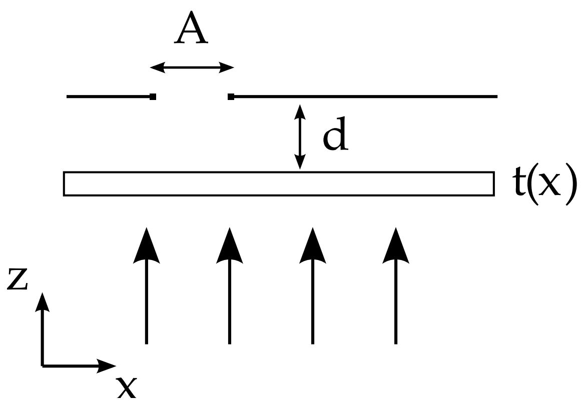
Λ<λ

$$k_z = \sqrt{k^2 - (k_x)^2} = \sqrt{(\frac{2\pi}{\lambda})^2 - (\frac{2\pi}{L})^2}$$



λ=532nm. Λ=250nm

Near-field imaging
NSOM



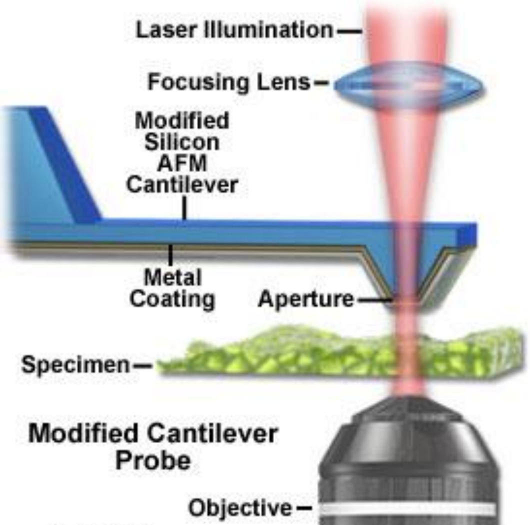
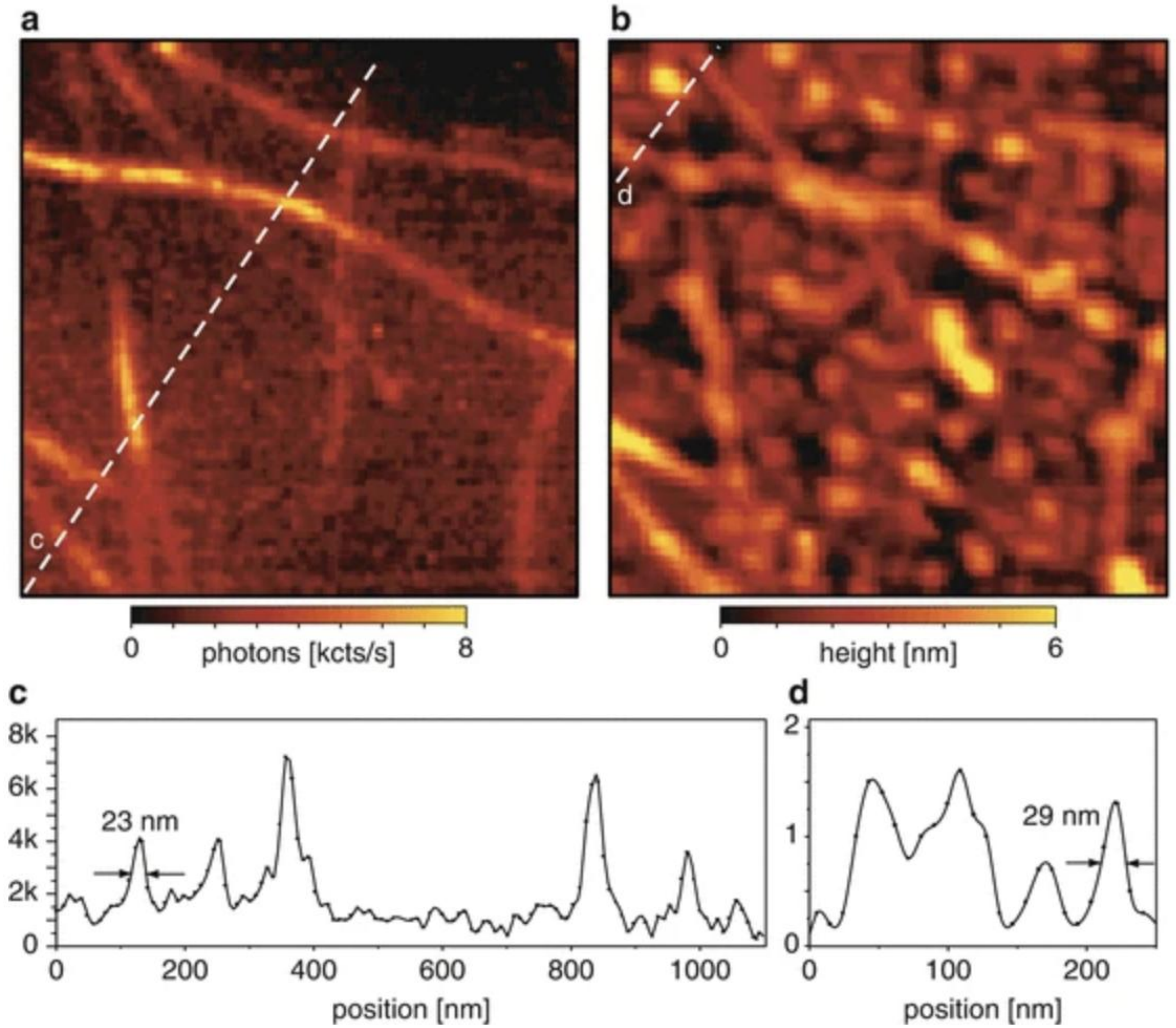


Figure 5



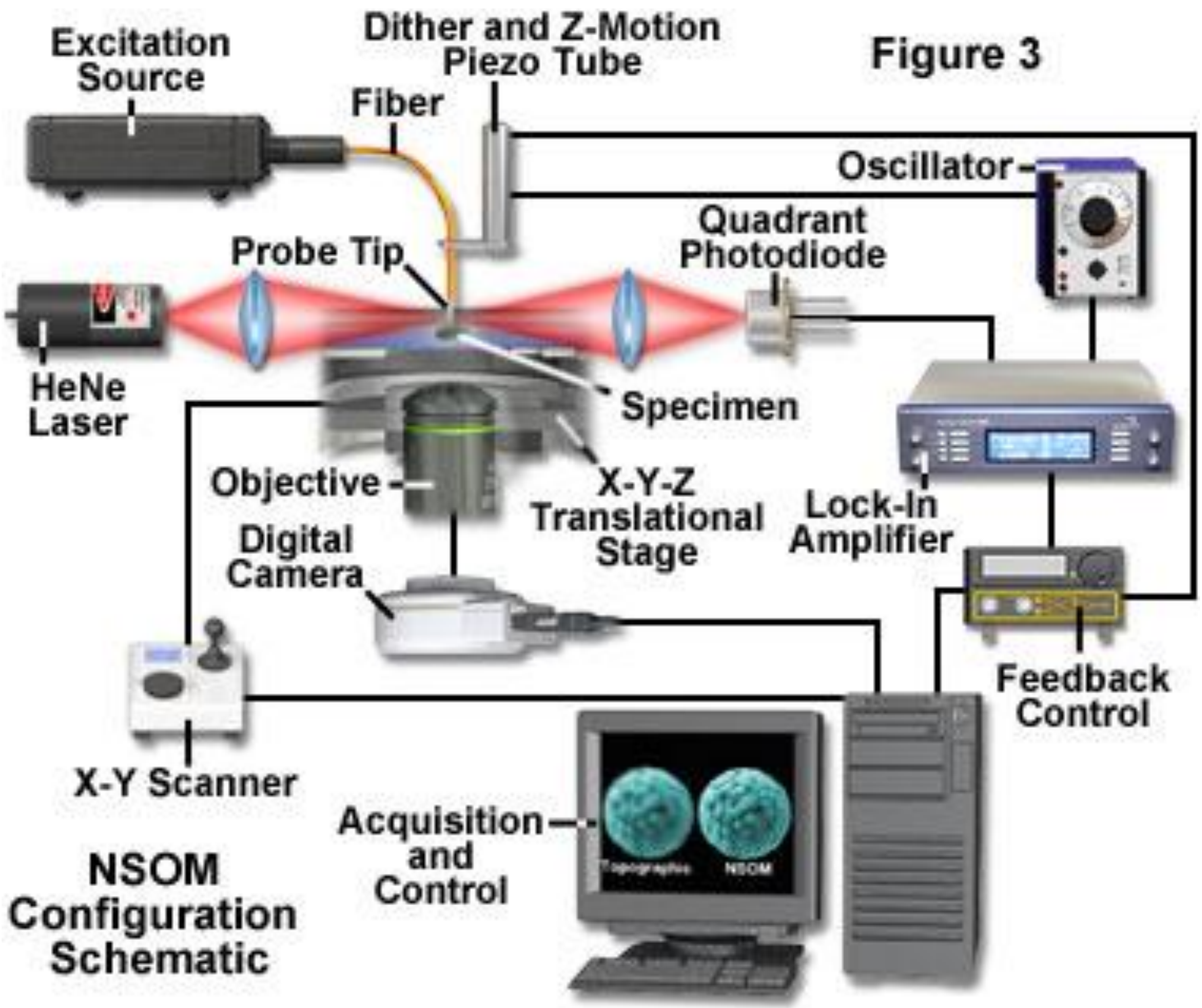
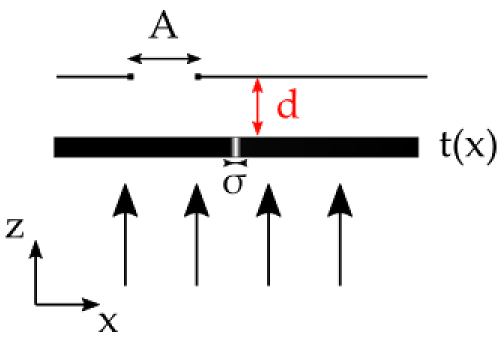
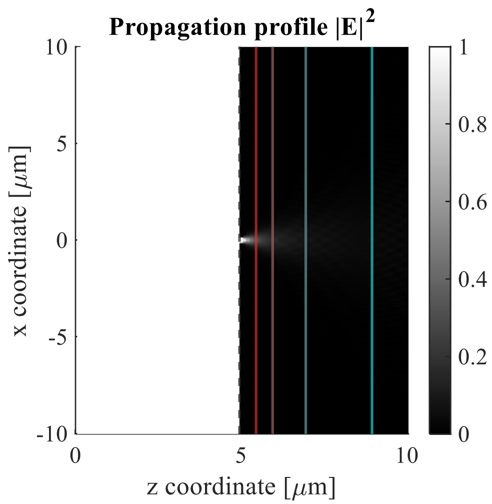


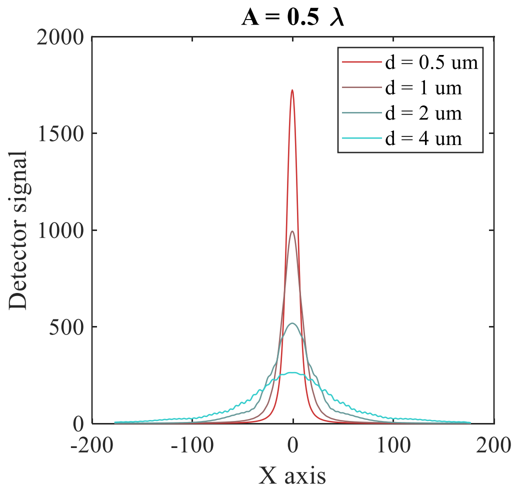
Figure 3
Excitation Source
Dither and Z-Motion Piezo Tube
Fiber
Oscillator
Quadrant Photodiode
Probe Tip
HeNe Laser
Specimen
Objective
X-Y-Z Translational Stage
Lock-In Amplifier
Digital Camera
Feedback Control
X-Y Scanner
Acquisition and Control
Topographic
NSOM
NSOM Configuration Schematic

Example 2: Point source

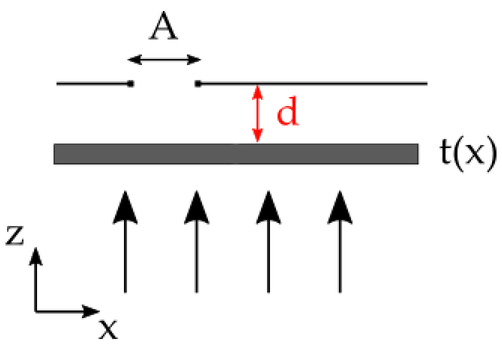
$\sigma = \lambda/2$

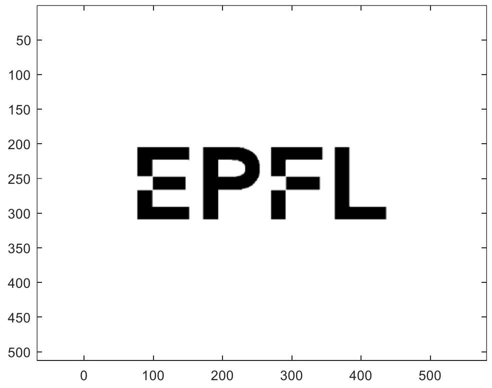


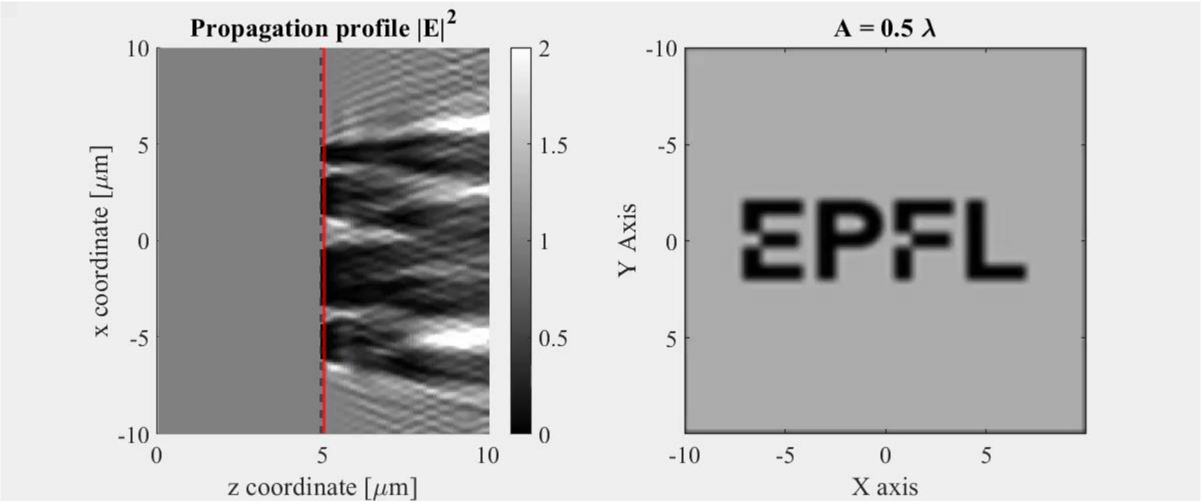




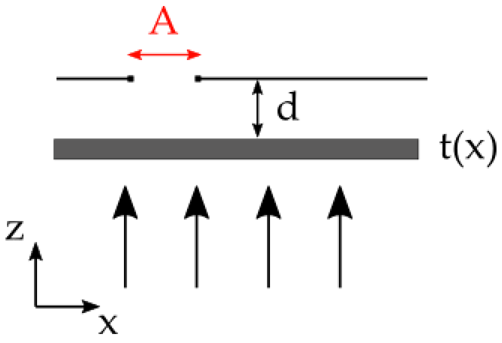
Example 4: Image

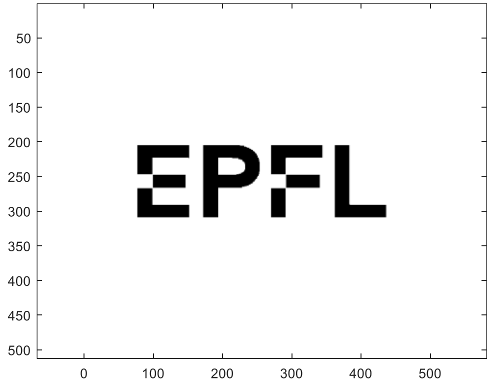


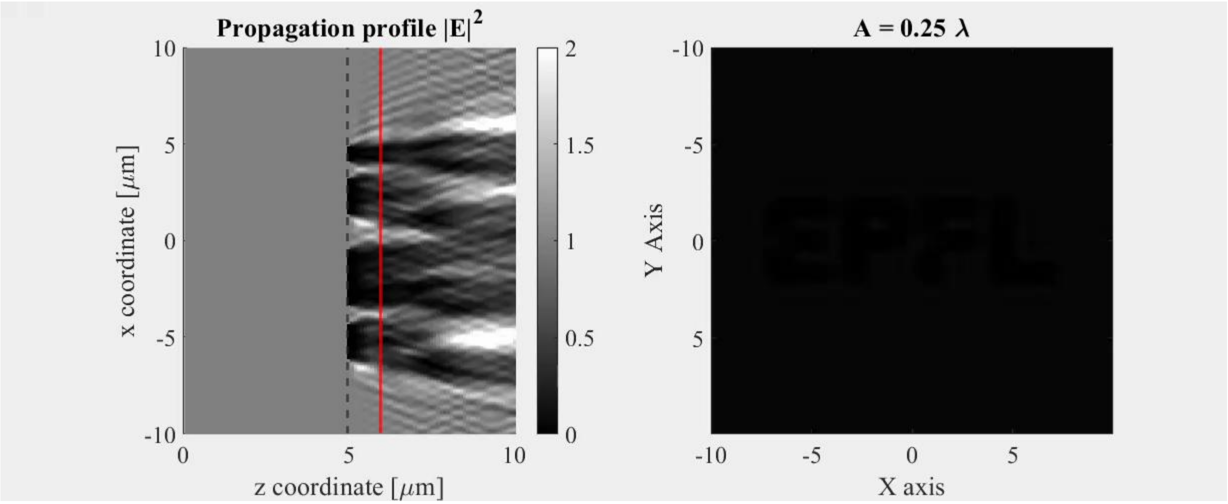




Example 4: Image

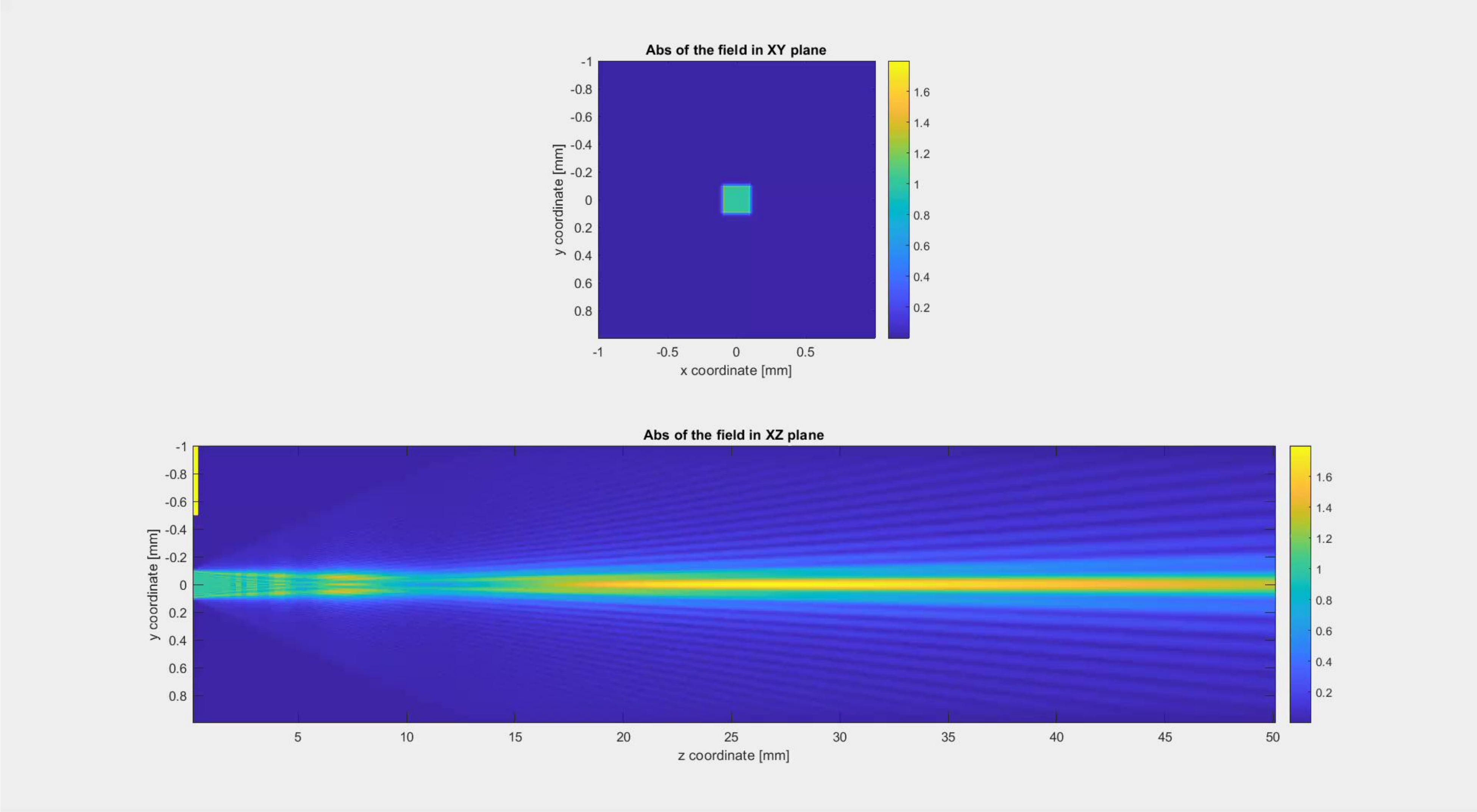






Fraunhofer Diffraction
or
Far field

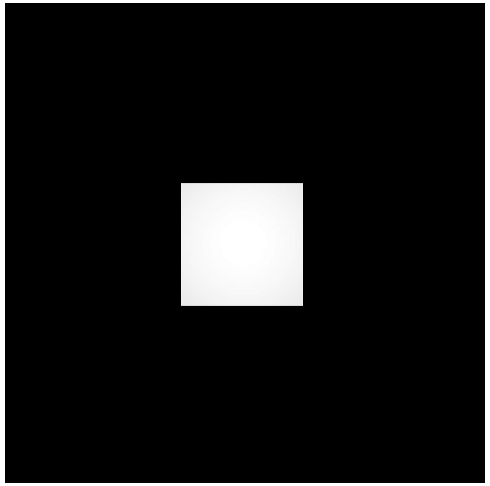
Propagation of square aperture



Square aperture - linear propagation

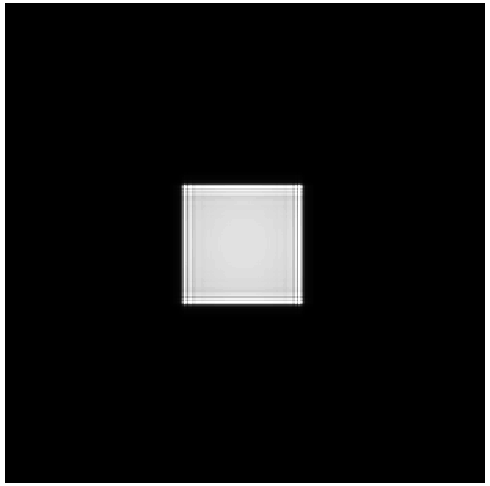
Aperture

z = 0mm



near field

z = 1mm



z = 20mm

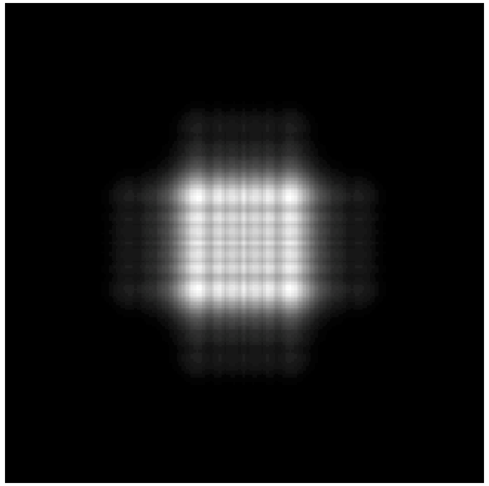
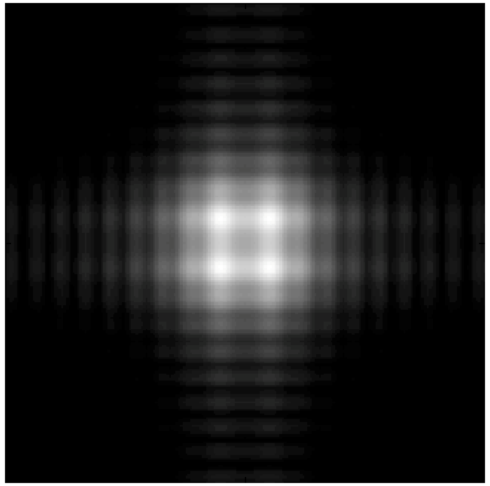
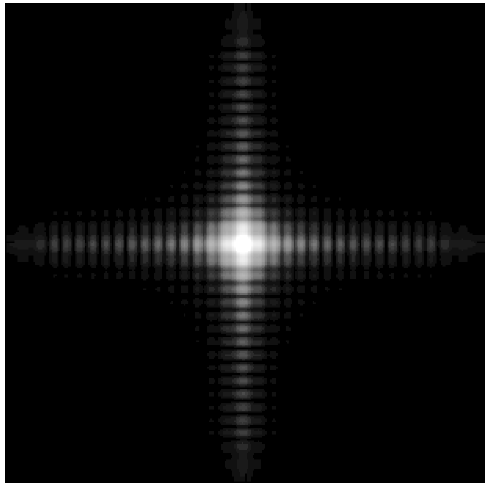


Image size = 2x2mm
Aperture side = 550µm
Wavelength = 800nm

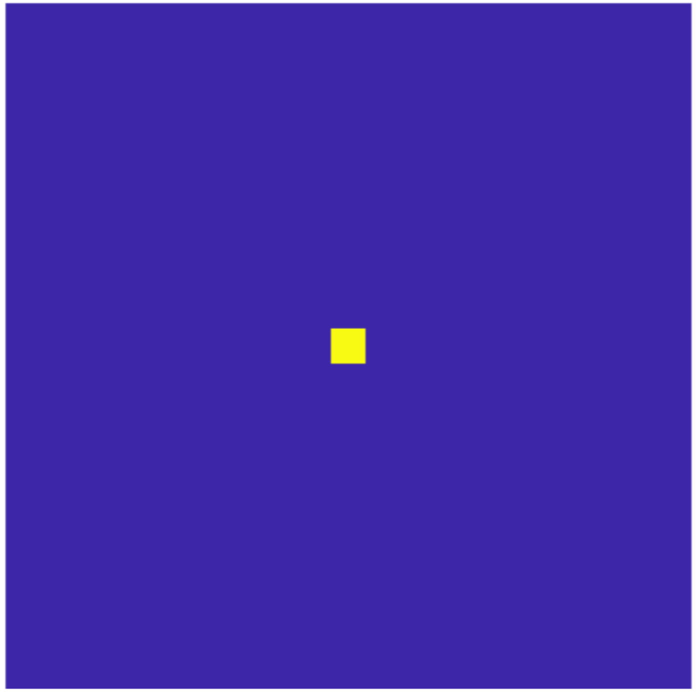
z = 100mm

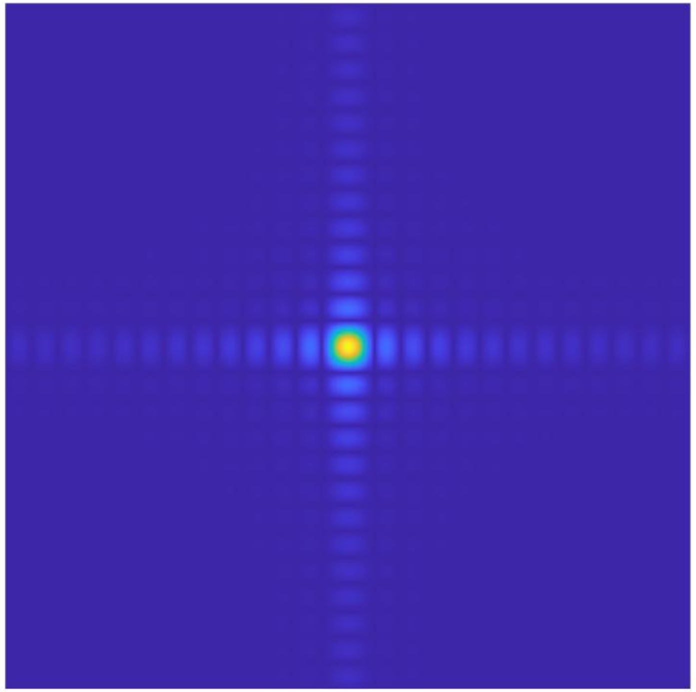


far field (rescaled)



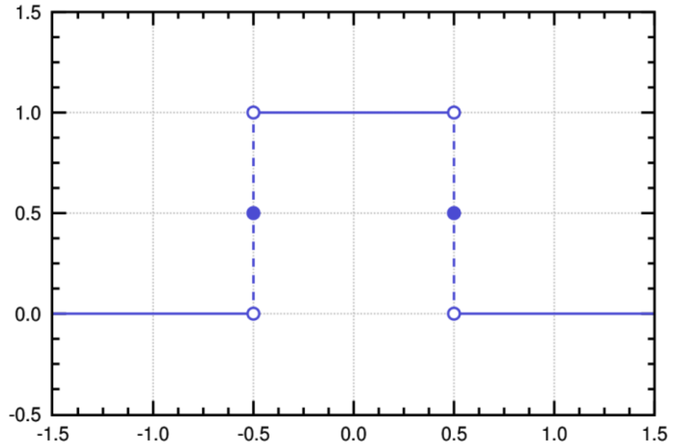
2D square and 2D sinc



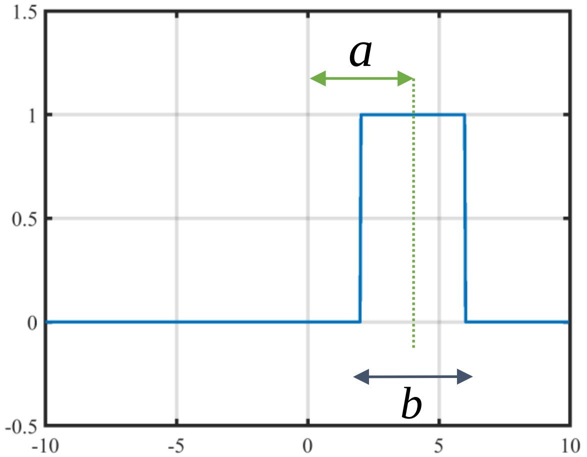


1D rect and 1D sinc

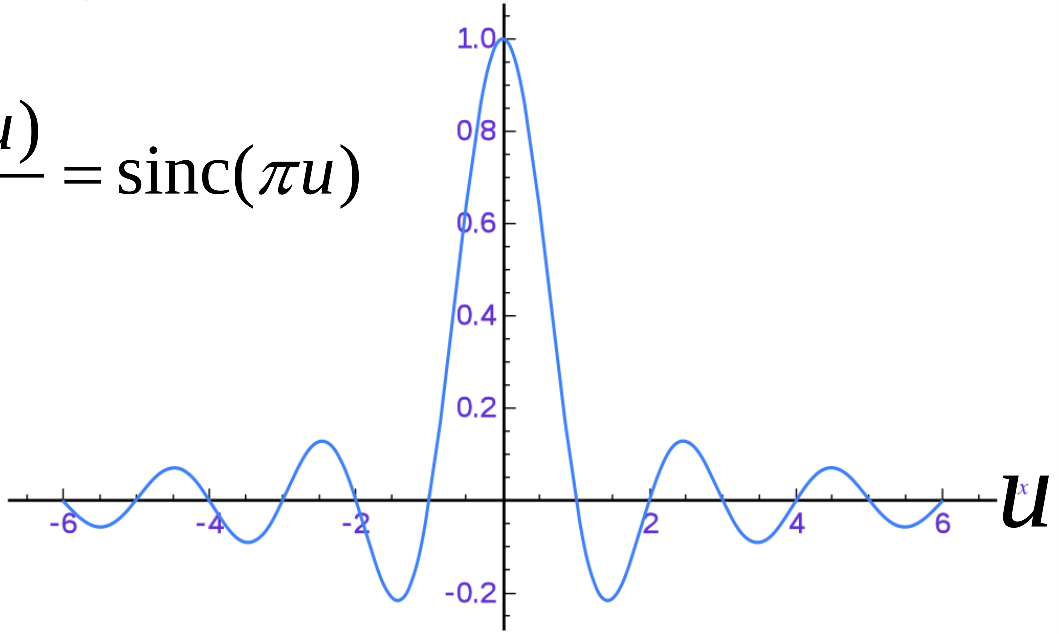
$$\mathrm{rect}(x) = \Pi(x) = \begin{cases} 0, & \text{if } |x| > 1/2 \\ 1/2, & \text{if } |x| = 1/2 \\ 1, & \text{if } |x| < 1/2 \end{cases}$$



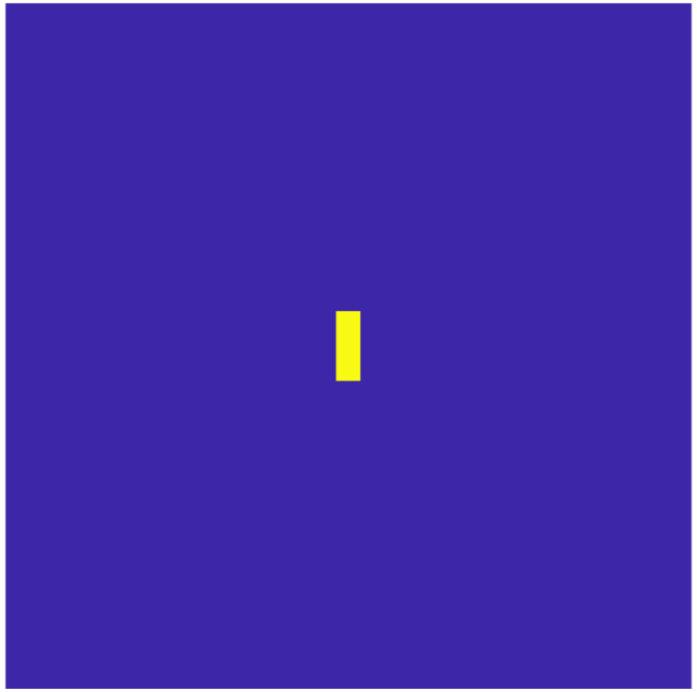
$$\mathrm{rect}\left[(x-a)/b\right]$$

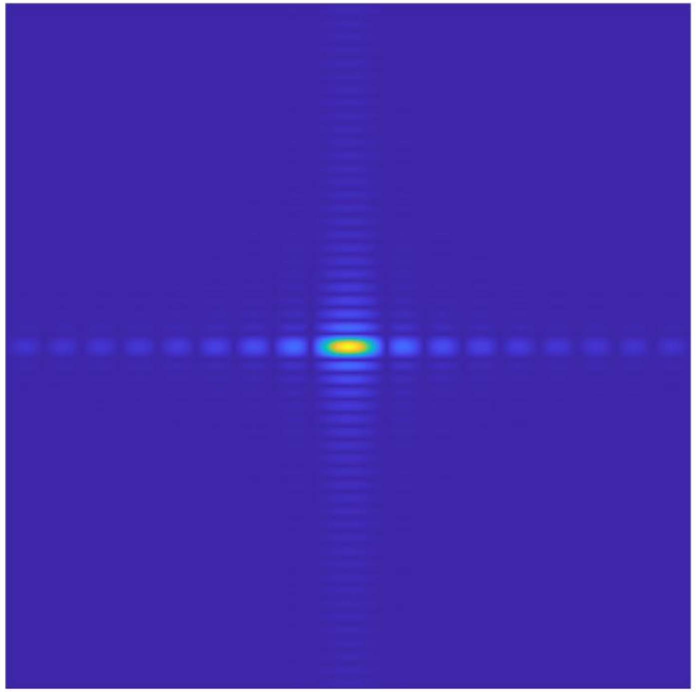


$$\int_{-\infty}^{\infty} \mathrm{rect}(x) e^{-j2\pi ux} dx = \frac{\sin(\pi u)}{\pi u} = \mathrm{sinc}(\pi u)$$



2D rectangle





Paraxial Approximation

$$e^{-j(k_x x + k_y y + k_z z)}$$

$$k_x^2 + k_y^2 + k_z^2 = k^2 = \left(\frac{2\pi}{\lambda}\right)^2$$

$$e^{-j(k_x x + k_y y + k_z z)} = e^{-j(k_x x + k_y y)} e^{-j\sqrt{k^2 - k_x^2 - k_y^2}\, z}$$

$$\cong e^{-j(k_x x + k_y y)} e^{-j\left(k - \frac{k_x^2 + k_y^2}{2k}\right) z} \quad \text{Note:} \sqrt{1 - x} \approx 1 - \frac{x}{2},\ x \ll 1$$

Paraxial Angular Spectrum

$$\boxed{\begin{aligned} &E(x,y,z \neq 0,t) \\ &= e^{j\omega t} \iint F(u,v)\, e^{-j\left(2\pi u x + 2\pi v y + \sqrt{k^2-(2\pi u)^2-(2\pi u)^2}\,z\right)}\, du\, dv \end{aligned}}$$

Paraxial plane wave:

$$\tilde{A}(k_x,k_y,z) = e^{j\left[\frac{k_x^2+k_y^2}{2k}\right]z}\,\tilde{A}(k_x,k_y,z=0)$$

Convolution Theorem:

$$g(x',y') = \int\int f(x,y)\,h(x'-x,y-y')\,dx\,dy$$

$$G(u,v) = F(u,v)\,H(u,v)$$

G, F and H are Fourier transforms of g, f and h respectively

Note that Fourier Transform of the exponential is:

$$e^{-\pi(x^2+y^2)} \Leftrightarrow e^{-\pi(u^2+v^2)} \quad \text{where} \quad k_x = 2\pi u \quad k_y = 2\pi v$$

Fresnel Diffraction

Let us apply the Convolution Theorem on Beam Propagation Equation:

$$A(x',y',z)=\frac{1}{j\lambda z}\iint A(x,y,z=0)\,e^{-\frac{j\pi[(x-x')^2+(y-y')^2}{\lambda z}}dxdy$$

Recall:

$$E_x\left(x,y,z,t\right)=A(x,y,z)e^{jWt}e^{-jkz}$$

Finally:

$$\boxed{E_x\left(x',y',z,t\right)=A(x',y',z)e^{j\omega t}e^{-jkz}=\frac{1}{j\lambda z}e^{j\omega t}e^{-jkz}\iint A(x,y,z=0)\,e^{-\frac{j\pi[(x-x')^2+(y-y')^2}{\lambda z}}dxdy}$$

Far Field

$$E_x\left(x',y',z,t\right) = A(x',y',z)e^{j\omega t}e^{-jkz}$$

$$= e^{j\omega t}e^{-jkz}\iint A(x,y,z=0)e^{-\frac{j\pi[(x-x')^2+(y-y')^2]}{\lambda z}}dxdy$$

$$= e^{j\omega t}e^{-jkz}e^{-\frac{j\pi(x'^2+y')^2}{\lambda z}}\iint A(x,y,z=0)e^{-\frac{j\pi(x^2+y)^2}{\lambda z}}e^{\frac{j2\pi(xx'+yy')}{\lambda z}}dxdy$$

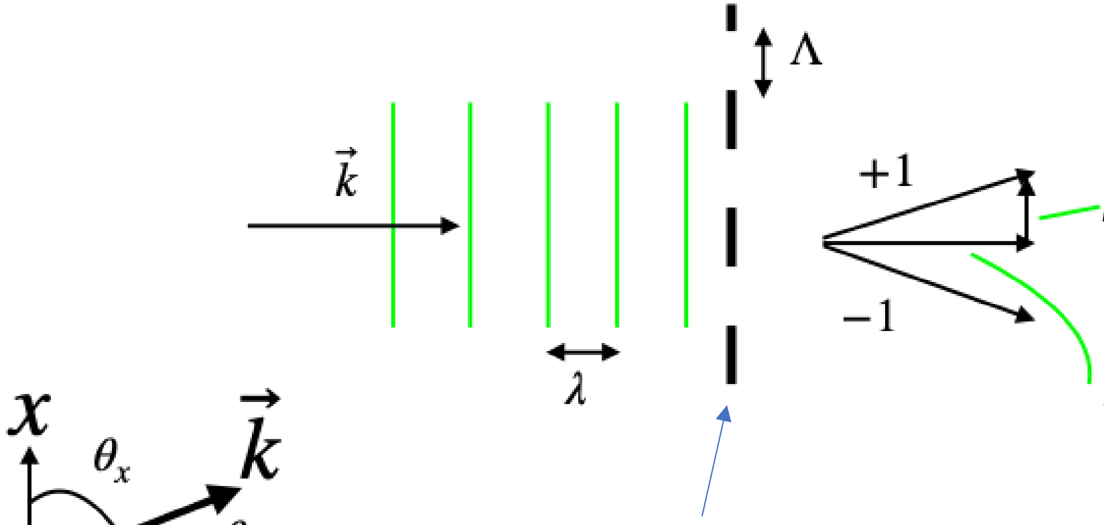
$$if\ \boxed{e^{-\frac{j\pi(x^2+y)^2}{\lambda z}} \approx 1}\ then\ A(x',y',z=0)\ is\ the\ FT\ of\ A(x,y,z)$$

$$u = x/\lambda z \quad v = y/\lambda z$$

Exercise #1

Grating

$$k_x^2 + k_y^2 + k_z^2 = \omega^2 \mu \epsilon = (\frac{2\pi}{\lambda})^2 = k^2 \qquad k_z = \sqrt{k^2 - (k_x^2 + k_y^2)}$$

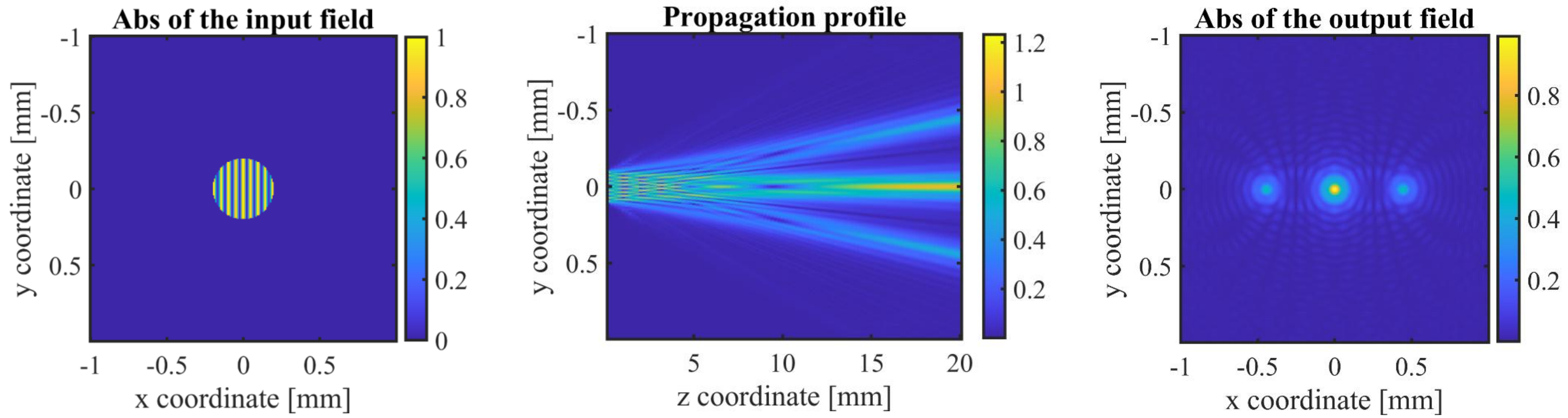


$$k_x = \frac{2\pi cos\theta_x}{\lambda} = \frac{2\pi}{\Lambda}$$

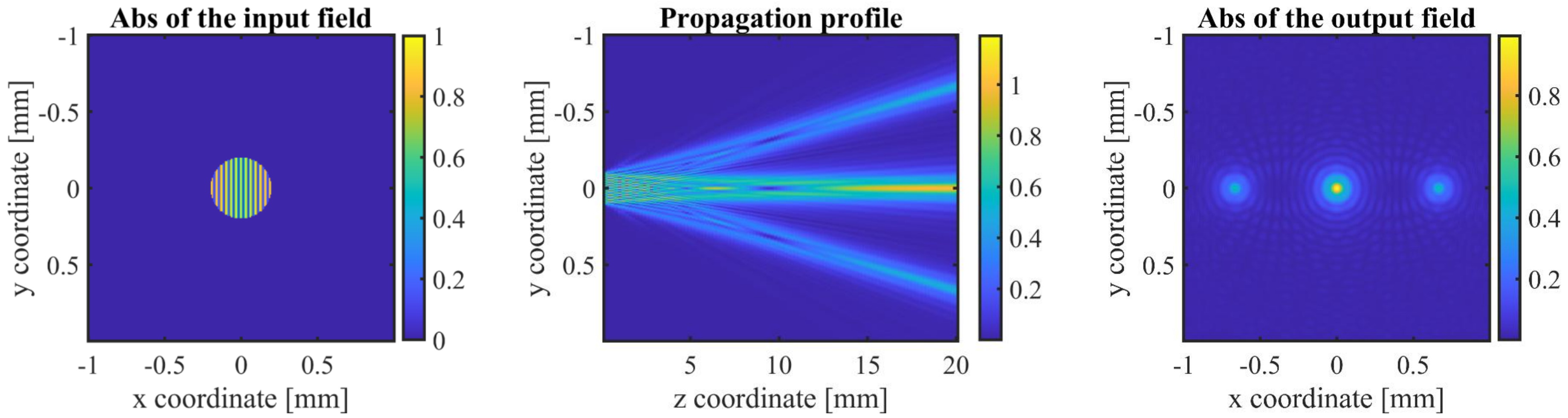
$$k_z = \frac{2\pi cos\theta_z}{\lambda} = \frac{2\pi}{\Lambda}$$

t(x)=$\frac{1}{2} + \frac{1}{2}$cos($\frac{2\pi}{\Lambda} x$)

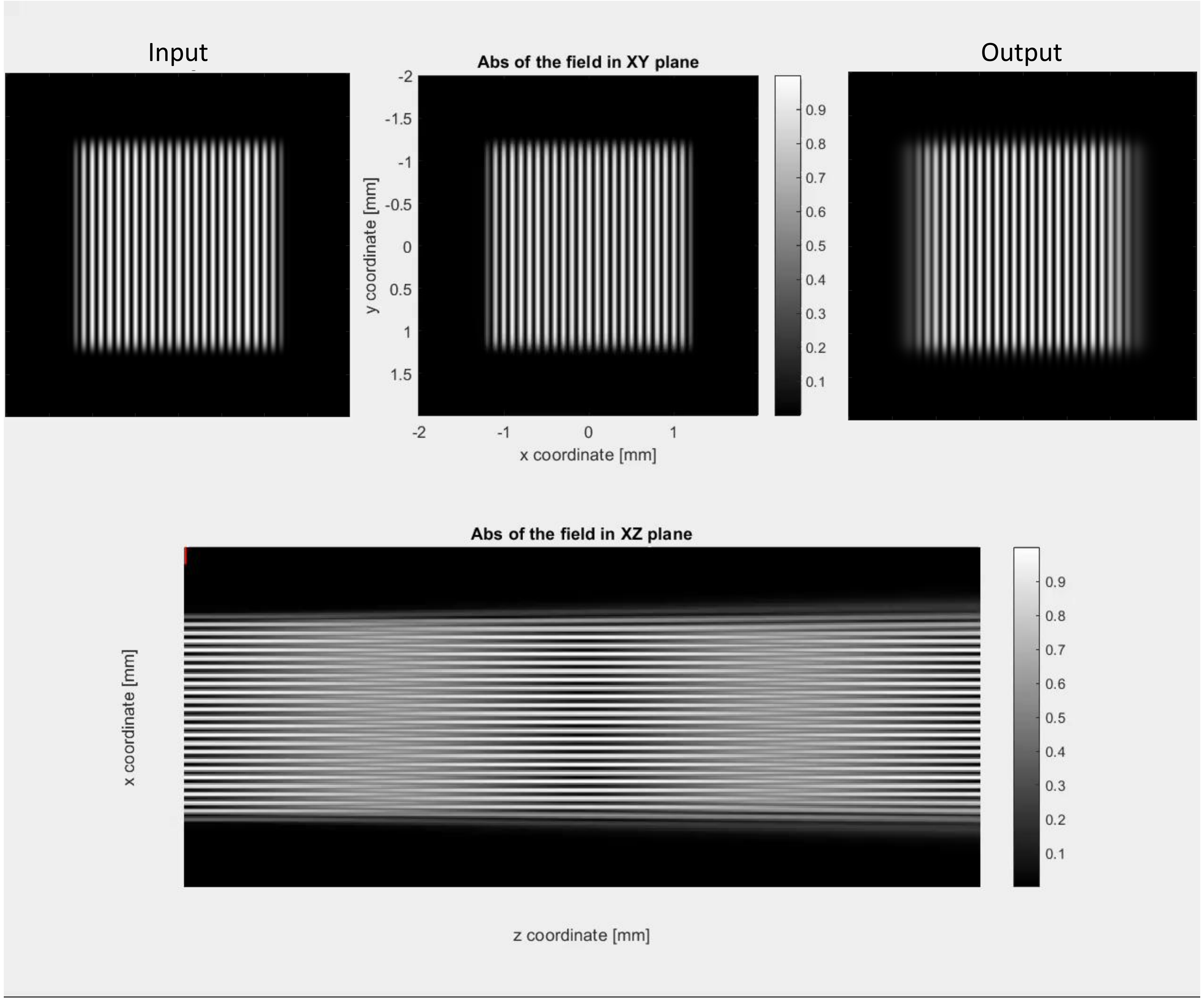
Period: 24 µm



Period: 16 µm



Talbot Effect



Talbot effect explanation

Angular Spectrum

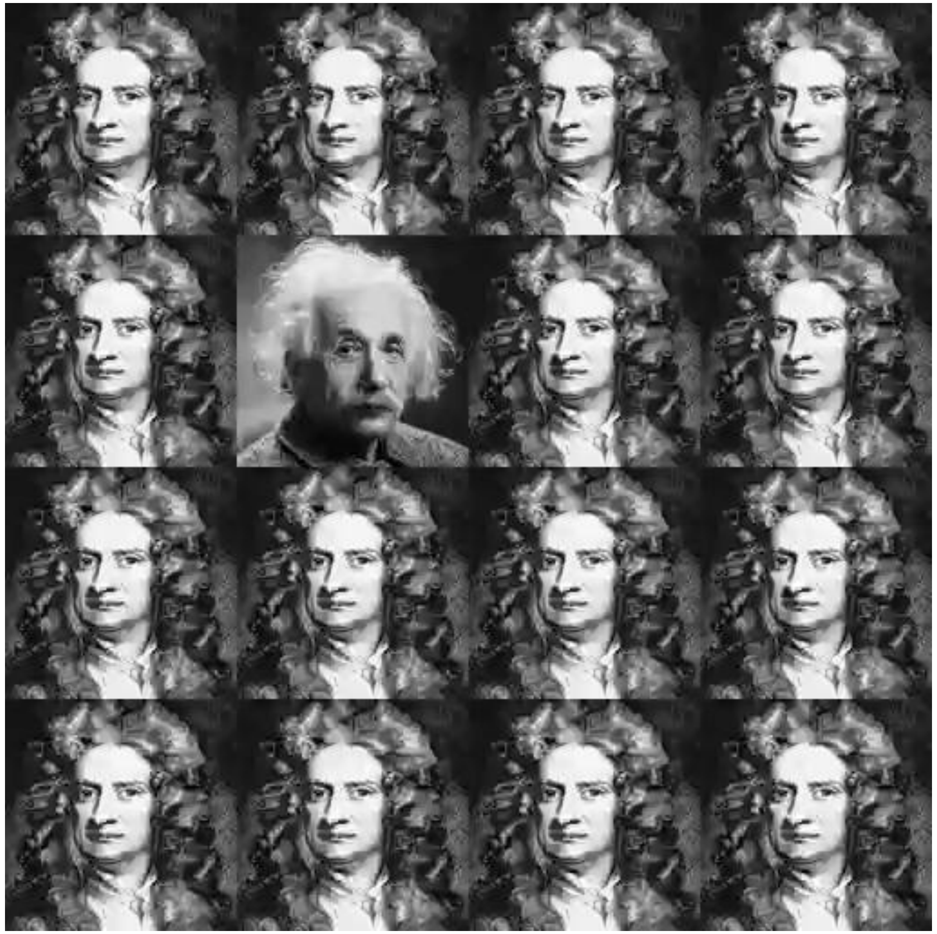
$$\boxed{\begin{aligned} &E(x,y,z \neq 0,t) \\ &= e^{j\omega t} \iint F(u,v) e^{-j\left(2\pi ux + 2\pi vy + \sqrt{k^2 - (2\pi u)^2 - (2\pi u)^2}\, z\right)} \, du\, dv \end{aligned}}$$

$$\text{for diffraction order } n: \quad \frac{k_x^2}{2k} z = \frac{(2\pi n/a)^2}{2(2\pi/\lambda)} z$$

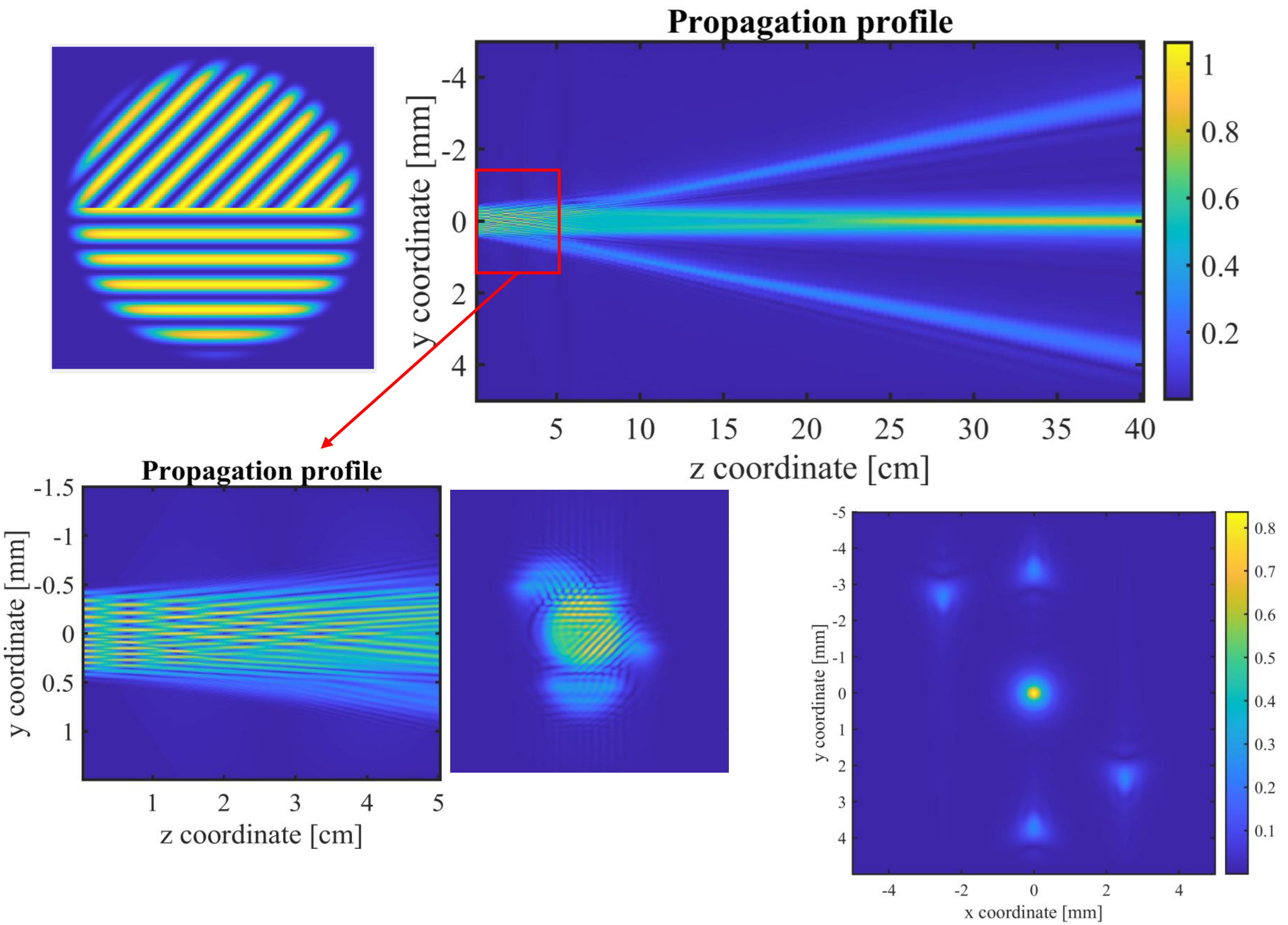
$$\text{for diffraction order } n+1: \quad \frac{k_x^2}{2k} z = \frac{(2\pi(n+1)/a)^2}{2(2\pi/\lambda)} z$$

$$\text{difference}: \frac{\left(\frac{2\pi}{a}\right)^2 [(n+1)^2 - n^2] z}{4\pi/\lambda} = \pi \frac{\lambda}{a^2} (2n+1) z$$

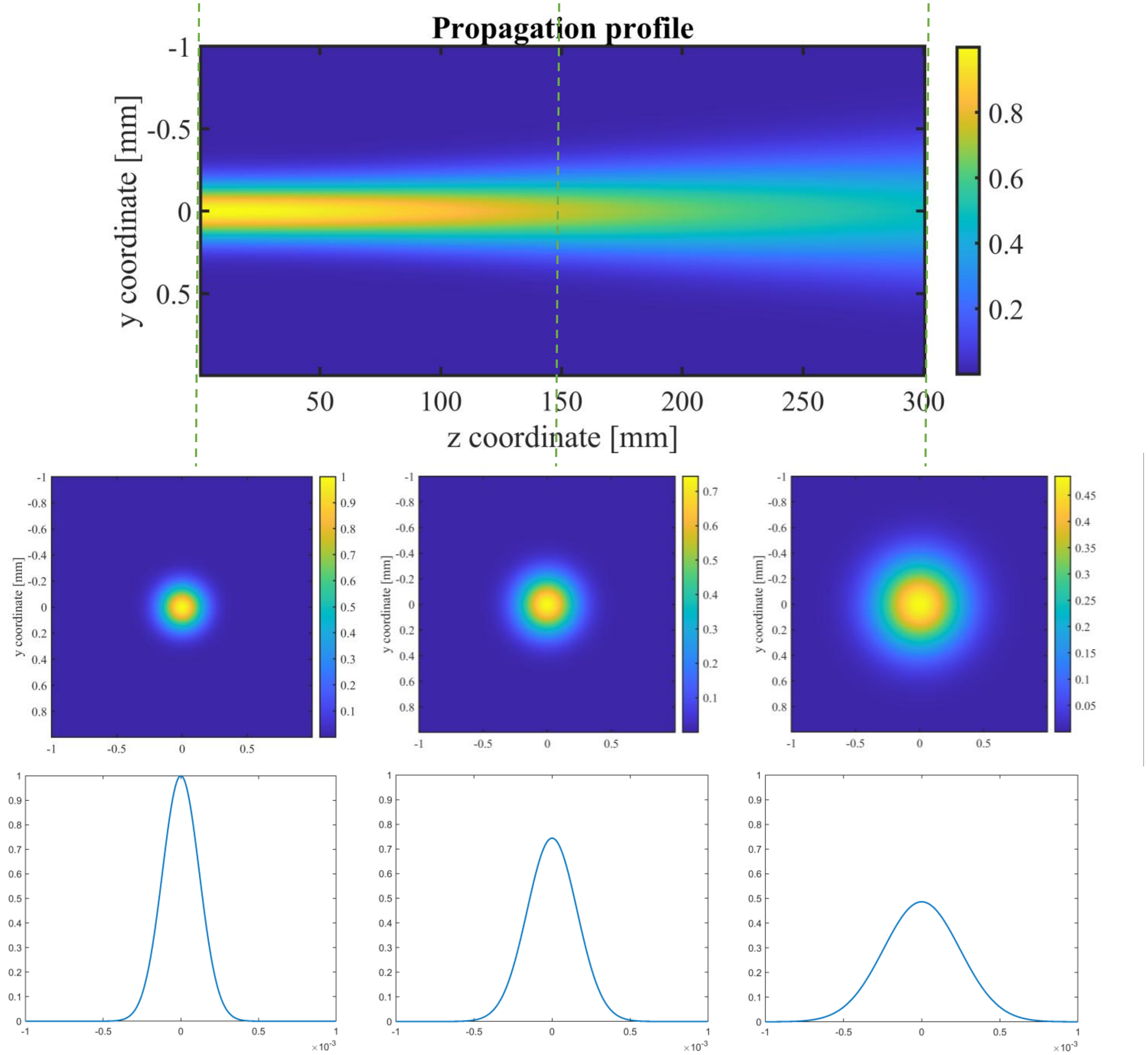
$$\Rightarrow \quad z_T = \frac{2a^2}{\lambda}$$



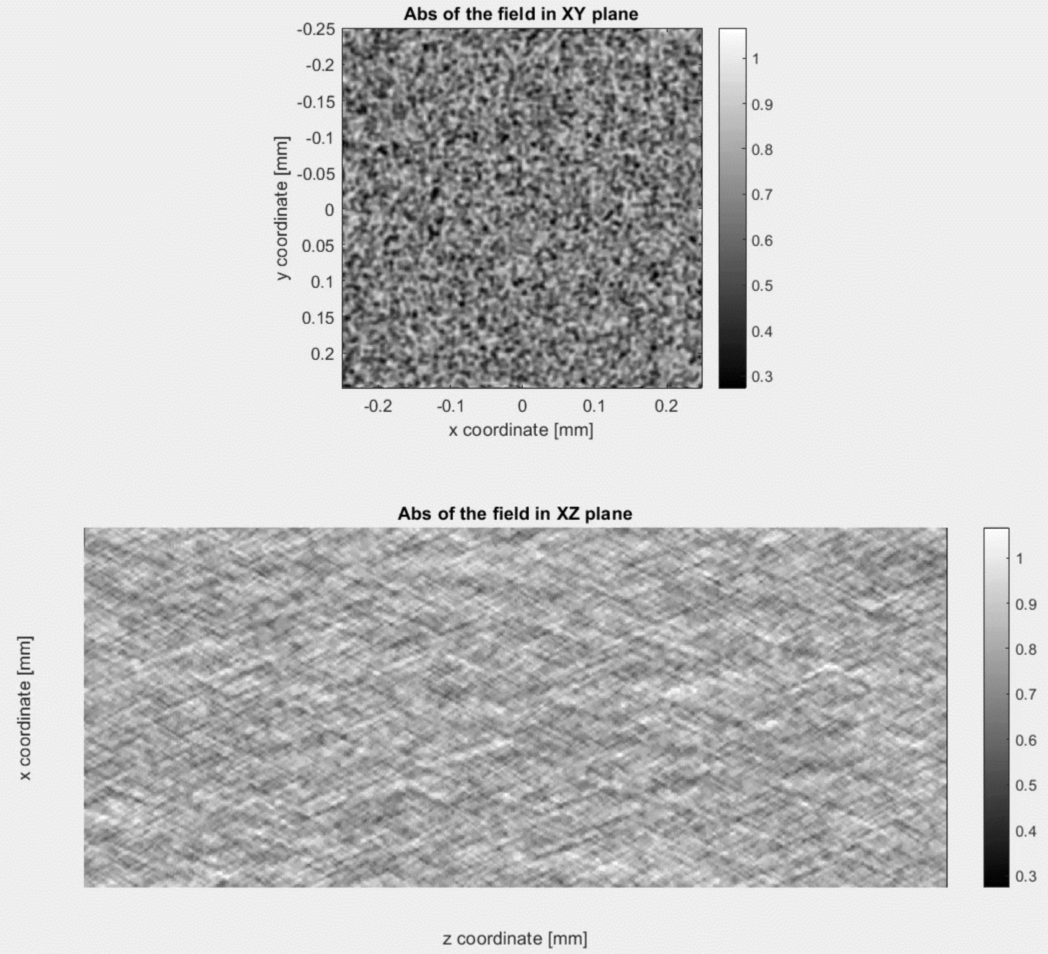
Approaching far field



Gaussian beam



Non-Magic Speckle



Magic speckle

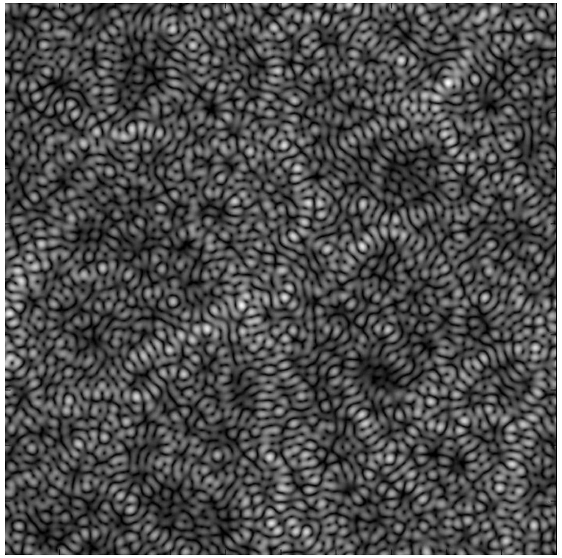
From the paraxial plane wave equation

$$\tilde{A}(2\pi u, 2\pi v, z) = e^{\frac{-j[(2\pi u)^2 + (2\pi v)^2]}{2k}z}\,\tilde{A}(2\pi u, 2\pi v, z=0)$$

$$\tilde{A}(2\pi u, 2\pi v, z) \approx \tilde{A}(2\pi u, 2\pi v, z=0) \text{ if } u^2 \lambda z \ll 1$$

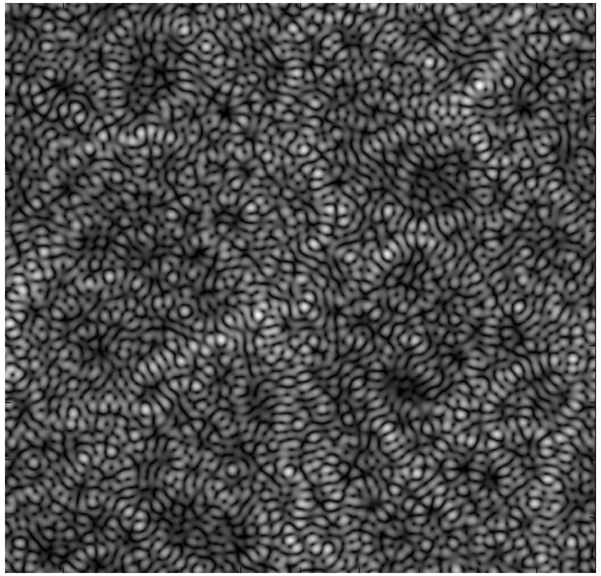
$$\text{if} \quad (2\pi u)^2 + (2\pi v)^2 = const$$

Magic Speckle input

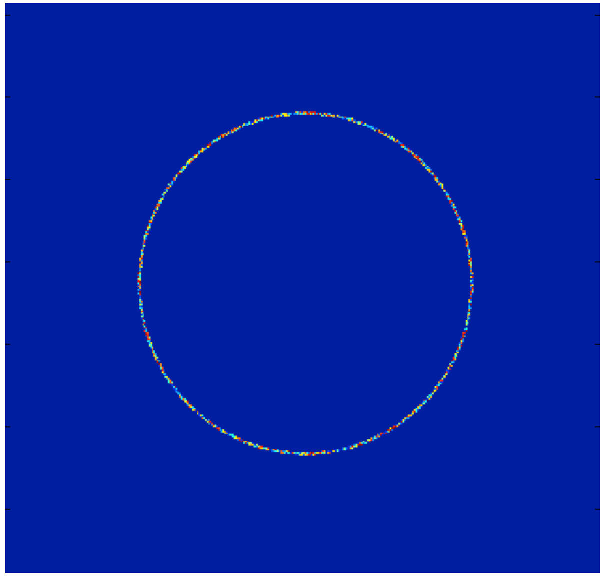


Why doesn't
It change??

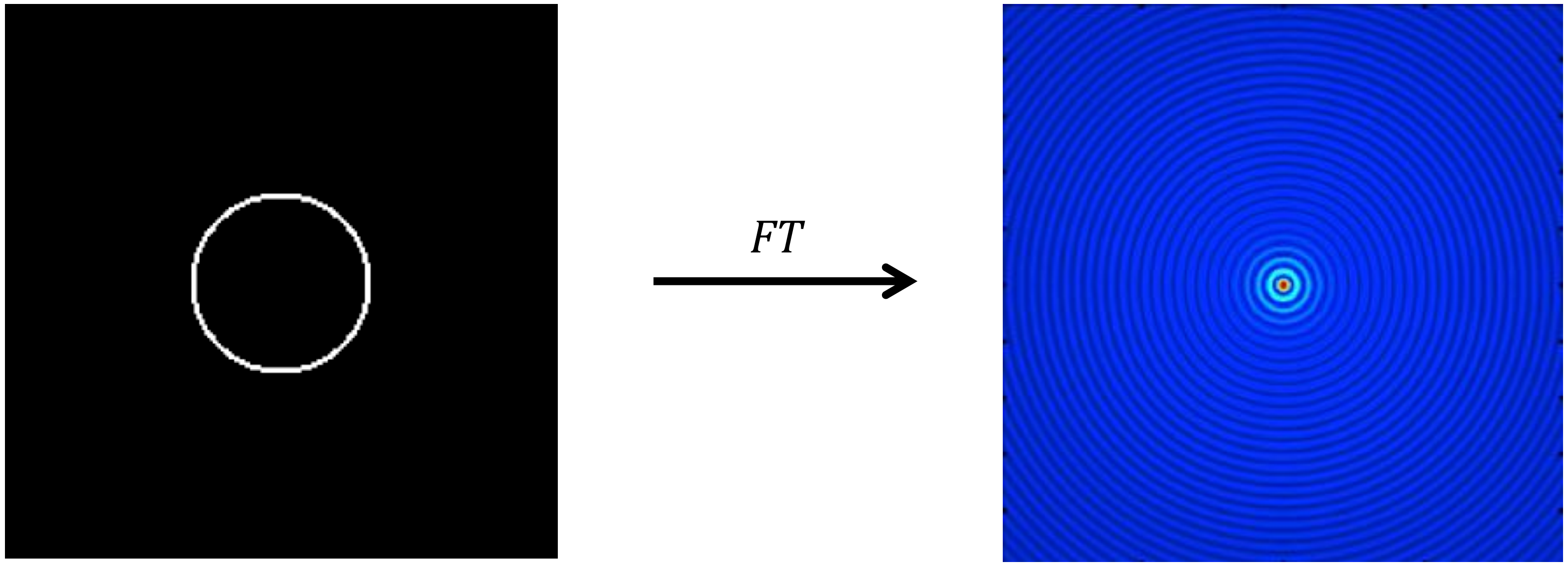
Magic Speckle input



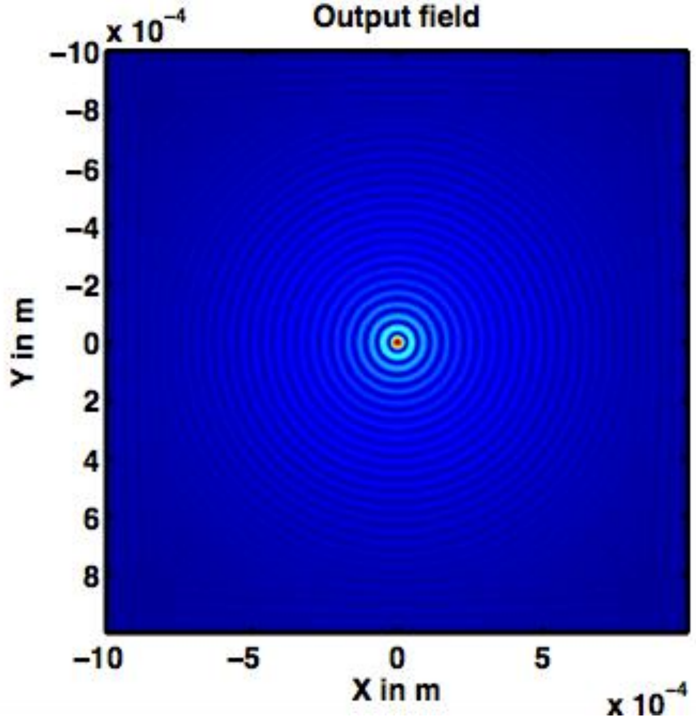
$\xrightarrow{FT}$

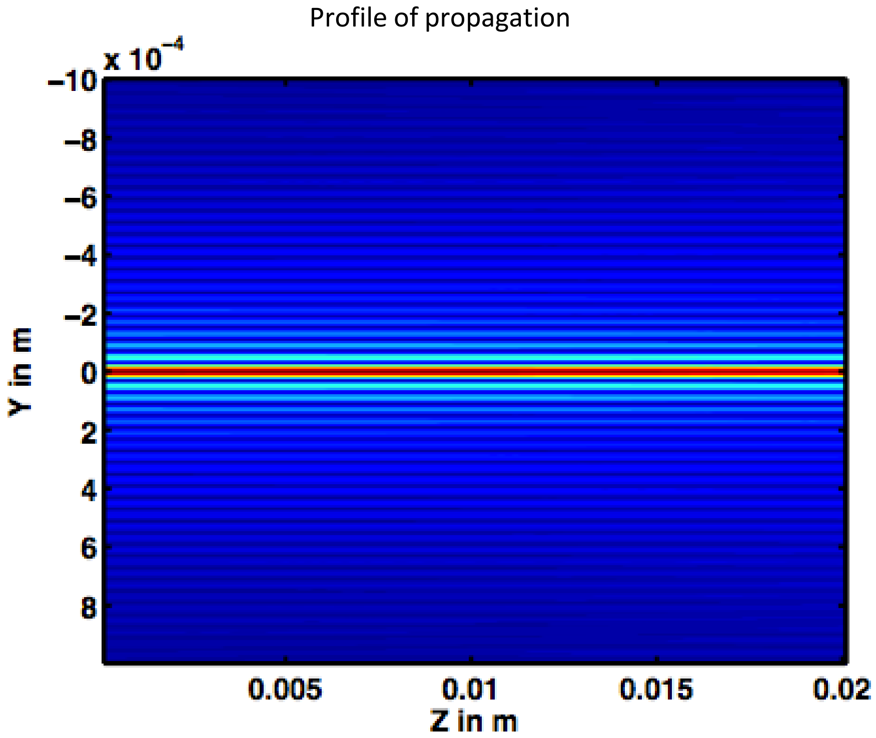


Bessel beam



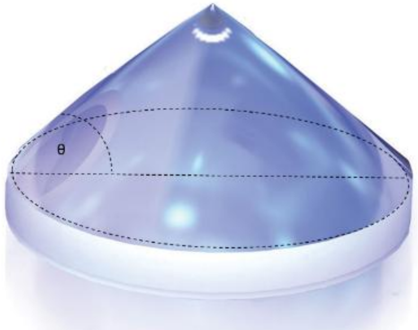
Propagation of a Bessel beam



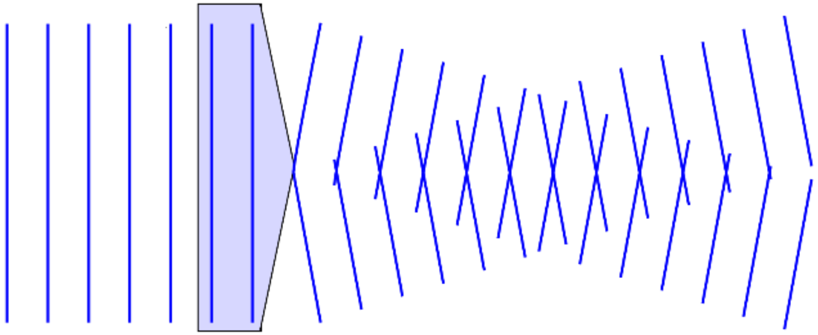


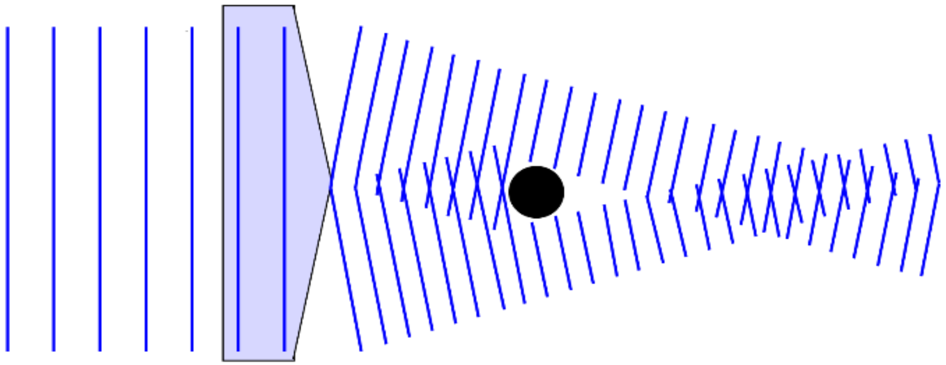
Almost no diffraction is observed after 2cm of propagation in free space

Self healing property



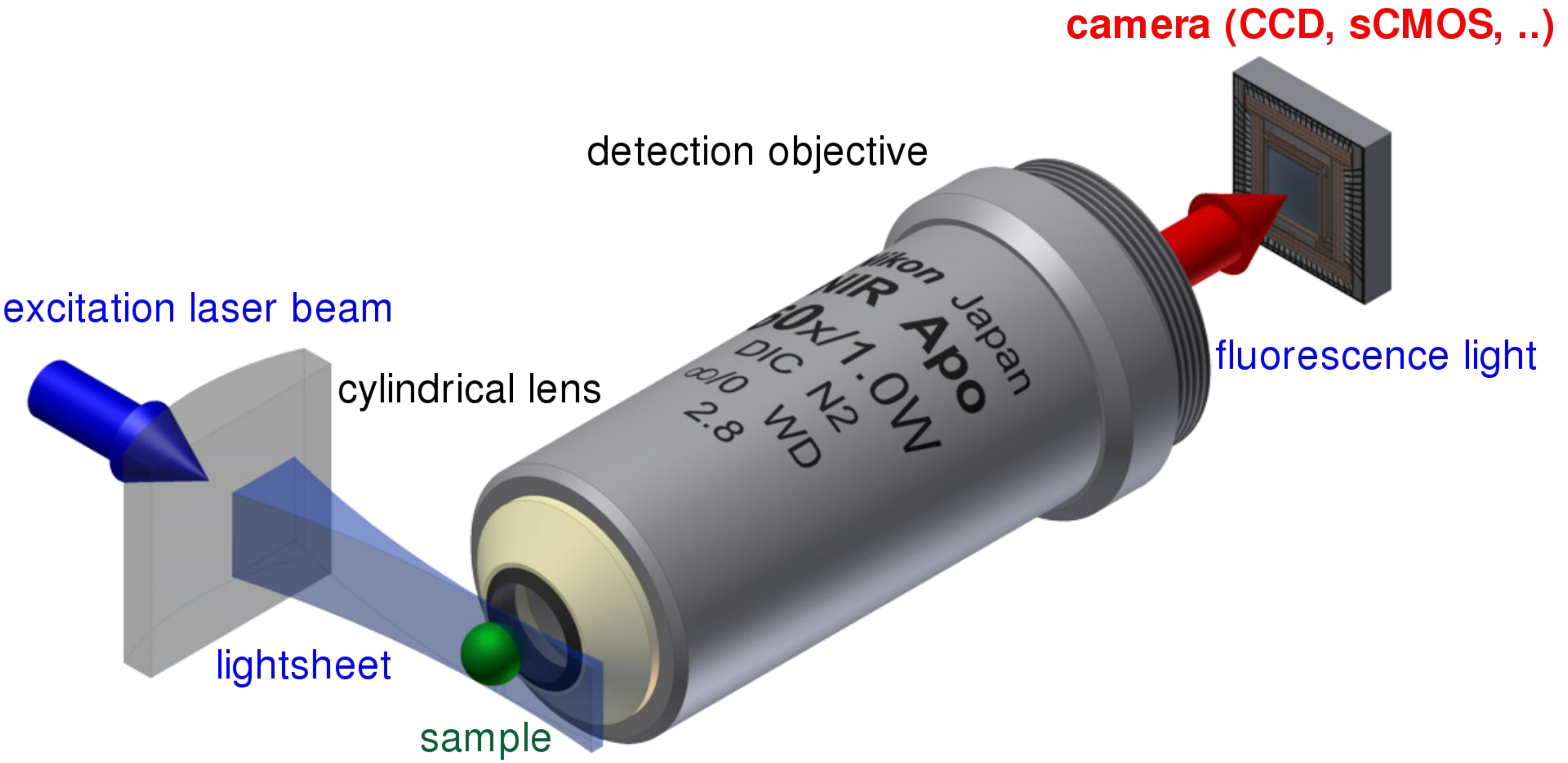
Axicon lens



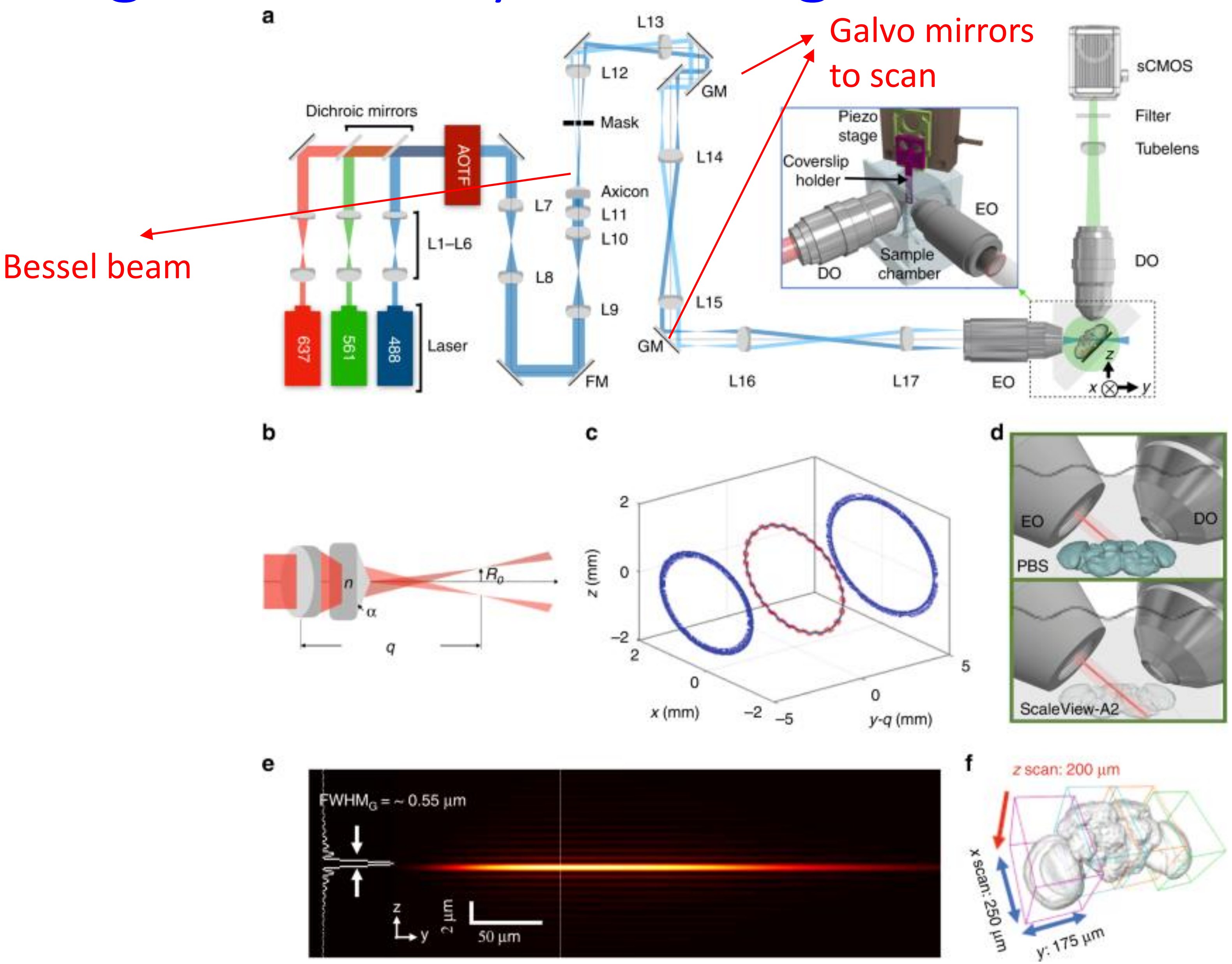


source: https://en.wikipedia.org/wiki/Bessel_beam

Light Sheet Imaging

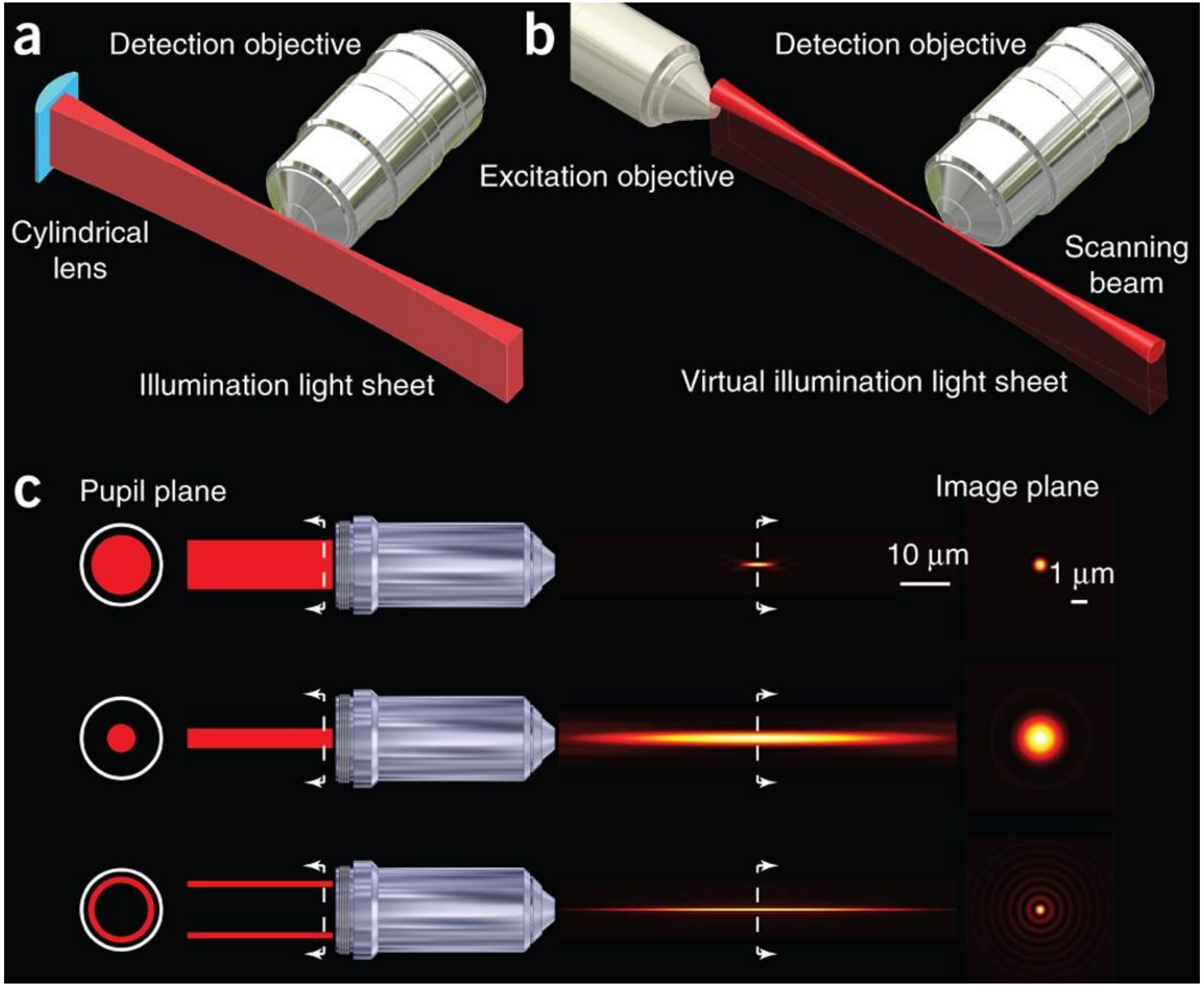


Light Sheet by scanning a Bessel beam



Chu, Li-An, et al. "Rapid single-wavelength lightsheet localization microscopy for clarified tissue." *Nature communications* 10.1 (2019): 1-10.

Light Sheet by scanning a Bessel beam



Gao, Liang, et al. "3D live fluorescence imaging of cellular dynamics using Bessel beam plane illumination microscopy." *Nature protocols* 9.5 (2014): 1083-1101.